

Meta-Stable Brane Configurations by Quartic Superpotential for Bifundamentals

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Abstract

The type IIA nonsupersymmetric meta-stable brane configuration consisting of three NS5-branes, D4-branes and anti-D4-branes where the electric gauge theory superpotential has a quartic term for the bifundamentals besides a mass term is constructed. By adding the orientifold 4-plane and 6-plane to this brane configuration, we also describe the intersecting brane configurations of type IIA string theory corresponding to the meta-stable nonsupersymmetric vacua of corresponding gauge theories.

1 Introduction

It has been found that the dynamical supersymmetry breaking in meta-stable vacua [1, 2] can occur in the $\mathcal{N} = 1$ supersymmetric gauge theory with massive fundamental flavors. The extra mass term for the quarks in the superpotential has led to the fact that some of the F-term equations cannot be satisfied and then the supersymmetry is broken. The meta-stable brane realizations of type IIA string theory have been studied in [3, 4, 5].

Recently it has been found in [6, 7] that other kinds of the type IIA nonsupersymmetric meta-stable brane configuration can be constructed by considering an additional quartic term for the quarks in the superpotential besides the mass term for the quarks. Geometrically, this extra deformation in the gauge theory corresponds to the rotation of D6-branes along the (45)-(89) directions in type IIA string theory realization. Classically there exist only supersymmetric ground states because due to the extra quartic term all the F-term equations are satisfied. By adding the orientifold 6-plane to this brane configuration [6], the brane configuration corresponding to the meta-stable nonsupersymmetric vacua of the supersymmetric unitary gauge theory with symmetric flavor as well as fundamental flavors is found [8]. For the antisymmetric flavor case, the corresponding meta-stable brane configuration is also described in [9].

On the other hand, the NS5-brane configuration in type IIA string theory where there exist two types of NS5-branes, i.e., NS5-brane(012345) and NS5'-brane(012389), preserves $\mathcal{N} = 2$ supersymmetry in four dimensions [10]. By adding D4-branes(01236) and anti-D4-branes($\overline{D4}$ -branes) into this system, the supersymmetry is broken [11]. As the distance between the two NS5'-branes becomes zero, this brane configuration with D4- and $\overline{D4}$ -branes can decay and the geometric misalignment between flavor D4-branes arises. Also the meta-stable vacua of [1] appear in some region of parameter space.

It is natural to ask what happens when some of the NS-branes in the brane configuration of [11] are rotated, as suggested in [6]? Recall that what Giveon and Kutasov did in [6, 7] is to rotate D6-branes with some angles, compared to the brane configuration of [3, 4, 5]. What we do in this paper is to rotate some of the NS-branes with some angles, compared to the brane configuration of [11].

One expects, in the gauge theory side, that the quartic term for the bifundamentals appears in the superpotential. First, we construct the meta-stable brane configuration by rotating the NS-brane in the brane configuration of [11] and secondly, we focus on the meta-stable brane configurations by adding an orientifold 4-plane and an orientifold 6-plane to this brane configuration, along the line of [3, 4, 5, 12, 13]. When the former is added, no extra

NS5-branes or D-branes are needed. However, when the latter is added, the extra NS5-branes or D-branes into the above brane configuration are needed in order to have a product gauge group. All of these examples have very simple dual magnetic superpotentials which make it easier to analyze the meta-stable brane configurations.

In section 2, we review the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1$ $SU(N_c) \times SU(N'_c)$ gauge theory with the bifundamentals and deform this theory by adding both the mass term and the quartic term for the bifundamentals. In the brane configuration, this is equivalent to a displacement and a rotation of NS5'-brane. Then we construct the dual magnetic theory which is $\mathcal{N} = 1$ $SU(\tilde{N}_c) \times SU(N'_c)$ gauge theory with corresponding dual matter as well as gauge singlet for the first gauge group factor. This corresponds to an interchange of two NS-branes. We consider the nonsupersymmetric meta-stable minimum and present the corresponding intersecting brane configurations of type IIA string theory. Some of the flavor D4-branes are approaching the NS5-brane.

In section 3, we review the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1$ $Sp(N_c) \times SO(2N'_c)$ gauge theory with a bifundamental and deform this theory by adding the mass term and the quartic term for the bifundamental. Due to the presence of O4-plane, a displacement and a rotation of NS5'-brane occur also for the mirror of NS5'-brane. Then we construct the dual magnetic theory which is $\mathcal{N} = 1$ $Sp(\tilde{N}_c) \times SO(2N'_c)$ gauge theory with corresponding dual matter as well as gauge singlet for the first gauge group factor. We consider the nonsupersymmetric meta-stable minimum and present the corresponding intersecting brane configurations of type IIA string theory. Detaching of flavor D4-branes happens also for the mirrors. We also discuss the dual magnetic theory which is $\mathcal{N} = 1$ $Sp(N_c) \times SO(2\tilde{N}'_c)$ gauge theory briefly. Contrary to the unitary case in section 2, the rank of gauge group and matter contents are different from the one in previous case.

In section 4, we describe the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N} = 1$ $SU(N_c) \times SU(N'_c)$ gauge theory with different matters and deform this theory by adding both the mass term and the quartic term for the bifundamentals. Due to the presence of O6-plane, a displacement and a rotation of NS5-brane occur also for the mirror of NS5-brane. Then we construct the dual magnetic theory which is $\mathcal{N} = 1$ $SU(\tilde{N}_c) \times SU(N'_c)$ gauge theory with corresponding dual matters as well as gauge singlet for the first gauge group factor. This corresponds to an interchange of two NS5'-branes. We consider the nonsupersymmetric meta-stable minimum and present the corresponding intersecting brane configurations of type IIA string theory. Detaching of flavor D4-branes happens for the mirrors but the O6-plane action behaves differently, compared with the one of O4-plane in section 3. We also consider the same gauge theory with other different matters

and describe the nonsupersymmetric meta-stable brane configuration from the dual magnetic theory which is $\mathcal{N} = 1$ $SU(\tilde{N}_c) \times SU(N'_c)$ gauge theory.

In section 5, we make some comments for the future directions after the summary of this paper.

2 Meta-stable brane configuration with three NS-branes

2.1 Electric theory

The type IIA brane configuration [14, 15] corresponding to $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group

$$SU(N_c) \times SU(N'_c) \quad (2.1)$$

and a bifundamental X in the representation $(\mathbf{N}_c, \overline{\mathbf{N}'_c})$ and its conjugate field \tilde{X} in the representation $(\overline{\mathbf{N}_c}, \mathbf{N}'_c)$, under the gauge group (2.1) can be described as follows: the middle NS5-brane(012345), the left $NS5'_L$ -brane(012389), the right $NS5'_R$ -brane(012389), N_c - and N'_c -color D4-branes(01236). We take the arbitrary number of color D4-branes with the constraint $N'_c \geq N_c$. The bifundamentals X and \tilde{X} correspond to 4-4 strings connecting the N_c -color D4-branes with N'_c -color D4-branes ¹.

The middle NS5-brane is located at $x^6 = 0$ and the x^6 coordinates for the $NS5'_L$ -brane and $NS5'_R$ -brane are given by $x^6 = -y_2$ and $x^6 = y_1$ respectively, along the line of [11]. The N_c D4-branes are suspended between the NS5-brane(whose x^6 coordinate is given by $x^6 = 0$) and $NS5'_R$ -brane(whose x^6 coordinate is given by $x^6 = y_1$) while the N'_c D4-branes are suspended between the $NS5'_L$ -brane(whose x^6 coordinate is given by $x^6 = -y_2$) and the NS5-brane.

We draw this brane configuration ² in Figure 1A for the vanishing mass for the bifundamentals [11, 18]. The gauge couplings of $SU(N_c)$ and $SU(N'_c)$ are given by

$$g_1^2 = \frac{g_s \ell_s}{y_1}, \quad g_2^2 = \frac{g_s \ell_s}{y_2}. \quad (2.2)$$

As y_2 goes to the infinity, the $SU(N'_c)$ gauge group becomes a global symmetry and the theory above leads to SQCD with the gauge group $SU(N_c)$ and N'_c flavors in the fundamental representation.

¹ See also the relevant works in [16, 17] for gauge theory analysis in the context of supersymmetric vacua and [18, 19, 20] for nonsupersymmetric meta-stable vacua in the product gauge group theory.

²This is equivalent to the reduced brane configuration of Figure 1 in [18] if we remove or ignore all the D6-branes completely.

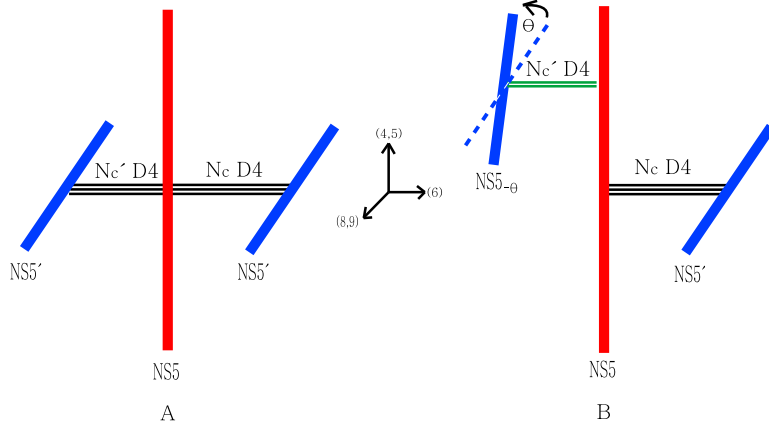


Figure 1: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $SU(N_c) \times SU(N'_c)$ and the bifundamentals X and \tilde{X} with vanishing mass term(1A) and non-vanishing mass and quartic terms(1B) for the bifundamentals. In Figure 1B, a “rotation” of $NS5'_L$ -brane in (w, v) -plane corresponds to a quartic term for the bifundamentals while a “displacement” of $NS5'_L$ -brane in $+v$ direction corresponds to a mass term for the bifundamentals. The superpotential for this brane realization of Figure 1B is given by (2.3) with the conditions (2.4).

According to the result of [18], there is no electric superpotential for the Figure 1A. Now let us deform this theory. Displacing the two $NS5'$ -branes relative each other in the

$$v \equiv x^4 + ix^5$$

direction [10] corresponds to turning on a quadratic superpotential for the bifundamentals X and \tilde{X} . Furthermore, rotating the $NS5'$ -branes in the (v, w) plane where we introduce [10]

$$w \equiv x^8 + ix^9$$

corresponds to turning on a quartic superpotential for the bifundamentals X and \tilde{X} [14, 21]. Let us denote them by $NS5_{L, -\theta_1}$ -brane and $NS5_{R, -\theta_2}$ -brane which are at angle $-\theta_1$ and $-\theta_2$ in (w, v) -plane respectively ³. Then the deformed electric superpotential is given by [14, 21]

$$W_{elec} = -\frac{\alpha}{2} \text{tr}(X\tilde{X})^2 + m \text{tr} X\tilde{X}, \quad \alpha = \frac{1}{\Lambda} (\tan \theta_1 + \tan \theta_2), \quad m = \frac{v_{NS5_{-\theta_1}}}{2\pi\ell_s^2}. \quad (2.3)$$

Here, the $NS5'_L$ -brane is moving to the $+v$ direction with N'_c D4-branes and the x^5 coordinate of $NS5'_L$ -brane is given by $+v_{NS5_{-\theta_1}}$. We focus on the case where

$$\theta_2 = 0 \quad \text{and} \quad \theta_1 \equiv \theta. \quad (2.4)$$

³The convention for $NS5_{-\theta}$ -brane here is different from the one in [22, 23] where the angle between unrotated $NS5'$ -brane and $NS5_{-\theta}$ -brane was not θ but $(\frac{\pi}{2} - \theta)$.

That is, the $NS5'_L$ -brane becomes $NS5_{L,-\theta}$ -brane and the $NS5'_R$ -brane remains $NS5_{R,-0} = NS5'_R$ -brane under the rotation. Giving an expectation value to the meson field $X\tilde{X}$ corresponds to recombination of N_c - and N'_c - color D4-branes in Figure 1A such that they are suspended between the $NS5'_L$ -brane and the $NS5'_R$ -brane and pushing N_c D4-branes into the w direction.

We draw the deformed brane configuration in Figure 1B for nonvanishing mass for the bifundamentals by both moving the $NS5'_L$ -brane with N'_c color D4-branes to the $+v$ direction and rotating it by an angle $-\theta$ in (w, v) -plane as we mentioned. Compared with the brane configuration of [11], the difference is coming from the rotation of $NS5'_L$ -brane. Of course, the $\theta = 0$ limit for the Figure 1B reduces to the brane realization of [11].

The solution for the supersymmetric vacua can be written as $X\tilde{X} = \frac{m}{\alpha}$ through the F-term conditions. This breaks the gauge group $SU(N_c) \times SU(N'_c)$ to $SU(N_c - k), SU(N'_c - k)$ and $U(k)$ [14]. When the middle NS5-brane moves to $+w$ direction, then the three NS-branes intersect in three points in (v, w) -plane. Then $(N_c - k)$ D4-branes are connecting between the middle NS5-brane and the $NS5'_R$ -brane. The $(N'_c - k)$ D4-branes are connecting between the $NS5_{-\theta}$ -brane and the middle NS5-brane. Finally, k D4-branes are connecting between the $NS5_{-\theta}$ -brane and the $NS5'_R$ -brane directly. The distance from k D4-branes to the middle NS5-brane can be read off from the trigonometric geometry and it is given by $w = v_{NS5_{-\theta}} \cot \theta$ [14].

2.2 Magnetic theory

Starting from the Figure 1B, we apply the Seiberg dual to the $SU(N_c)$ factor in (2.1), and the middle NS5-brane and the right $NS5'_R$ -brane are interchanged each other. Then the number of color \tilde{N}_c was given by $\tilde{N}_c = N'_c - N_c$ connecting the $NS5'_R$ -brane and the NS5-brane from [10, 18]. By moving the NS5-brane in Figure 1B to the right all the way past the $NS5'_R$ -brane, one obtains the Figure 2A. Before arriving at the Figure 2A, there exists an intermediate step where the N'_c D4-branes are connecting between the $NS5_{-\theta}$ -brane and $NS5'_R$ -brane and \tilde{N}_c D4-branes are connecting between $NS5'_R$ -brane and NS5-brane. By introducing N'_c D4-branes and N'_c anti-D4-branes between $NS5'_R$ -brane and NS5-brane, reconnecting the former with the N'_c D4-branes that are connecting between the $NS5_{-\theta}$ -brane and the $NS5'_R$ -brane and moving those combined D4-branes to $+v$ -direction, one gets the final Figure 2A where we are left with $(N'_c - \tilde{N}_c)$ anti-D4-branes between $NS5'_R$ -brane and NS5-brane.

The dual gauge group is given by

$$SU(\tilde{N}_c = N'_c - N_c) \times SU(N'_c). \quad (2.5)$$

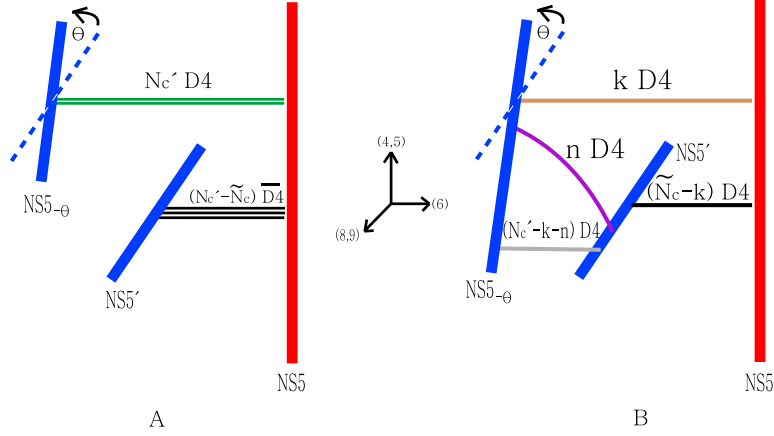


Figure 2: The magnetic brane configuration corresponding to Figure 1B with D4- and $\overline{D4}$ -branes(2A) when the distance between $NS5_{-\theta}$ -brane and the $NS5'_R$ -brane along v direction is large and with a misalignment between D4-branes(2B) when they are close to each other. At first, the N'_c flavor D4-branes connecting between $NS5_{-\theta}$ -brane and $NS5'_R$ -brane are splitting into $(N'_c - k)$ and k D4-branes. The location of intersection between $NS5_{-\theta}$ -brane and $(N'_c - k)$ D4-branes is given by $(v, w) = (0, +v_{NS5_{-\theta}} \cot \theta)$ while the one between $NS5_{-\theta}$ -brane and k D4-branes is given by $(v, w) = (+v_{NS5_{-\theta}}, 0)$. Secondly, by moving n flavor D4-branes from $(N'_c - k)$ flavor D4-branes, the nonzero positive w coordinate for n “curved” flavor D4-branes is determined later.

The matter contents are the field Y in the bifundamental representation $(\tilde{\mathbf{N}}_c, \overline{\mathbf{N}}'_c)$, its complex conjugate field \tilde{Y} in the bifundamental representation $(\overline{\mathbf{N}}_c, \mathbf{N}'_c)$, under the dual gauge group (2.5) and the gauge singlet $M \equiv X\tilde{X}$ in the representation for $(\mathbf{1}, \mathbf{N}'^2_c - \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1})$ under the dual gauge group. A cubic superpotential arises as an interaction between dual “quarks” Y, \tilde{Y} and a meson M . The low energy dynamics of the magnetic brane configuration can be described by the $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group (2.5) and the gauge couplings for the two gauge group factors are given by

$$g_{1,mag}^2 = \frac{g_s \ell_s}{y_1}, \quad g_{2,mag}^2 = \frac{g_s \ell_s}{y_2 - y_1}.$$

Then the dual magnetic superpotential, by adding the mass term for the bifundamentals X, \tilde{X} (which can be interpreted as a linear term in the meson M) and the quartic term for the bifundamentals X, \tilde{X} (that is a mass term in the meson M) to the above cubic superpotential, is given by

$$W_{dual} = \frac{1}{\Lambda} M Y \tilde{Y} - \frac{\alpha}{2} M^2 + m M. \quad (2.6)$$

The brane configuration for zero mass for the bifundamentals can be obtained from Figure 2A by pushing the $NS5_{-\theta}$ -brane together with N'_c D4-branes into the origin $v = 0$. Then the

number of dual colors for D4-branes becomes N'_c between the $NS5_{-\theta}$ -brane and the $NS5'_R$ -brane and \tilde{N}_c between the $NS5'_R$ -brane and the NS5-brane. Further zero limit of quartic term for the bifundamentals can be achieved by taking $\theta \rightarrow 0$ for the $NS5_{-\theta}$ -brane superpotential (2.6).

The conditions $b_{SU(\tilde{N}_c)}^{mag} = 2N'_c - 3N_c < 0$ and $b_{SU(N_c)} = 3N_c - N'_c > 0$ imply that $N'_c < \frac{3}{2}N_c$. Then the range for the N'_c can be written as $N_c < N'_c < \frac{3}{2}N_c$. Moreover, $b_{SU(N'_c)} = 3N'_c - N_c > 0$ and $b_{SU(N'_c)}^{mag} = N_c + N'_c > 0$. At the scale Λ_1 , the $SU(N_c)$ theory is strongly coupled and the Seiberg duality occurs. All the running couplings are changed by this duality and the coefficients of beta function $b_{SU(\tilde{N}_c)}^{mag}$ becomes negative and $b_{SU(N'_c)}^{mag}$ becomes positive. Then at energy scale lower than Λ_1 , the theory is weakly coupled. It is not enough to choose it lower than Landau pole Λ_1 simply because one cannot ignore the contributions from the coupling of $SU(N'_c)^{mag}$. Then under the constraint, $\Lambda_2 \ll \left(\frac{\Lambda_1}{\mu}\right)^b \Lambda_1 \ll \Lambda_1$ where $b \equiv \frac{b_{SU(\tilde{N}_c)}^{mag} - b_{SU(N'_c)}^{mag}}{b_{SU(N'_c)}}$, one can ignore the contribution from the gauge coupling of $SU(N'_c)^{mag}$ at the supersymmetry breaking scale and one relies on the one loop computation. Then one can use the magnetic superpotential (2.6) safely. See the appendix B of [24] for the relevant discussions and details.

The brane configuration in Figure 2A is stable as long as the distance $v_{NS5_{-\theta}}$ between the $NS5_{-\theta}$ -brane and the $NS5'_R$ -brane is large, as in [11, 6]. If they are close to each other, then this brane configuration is unstable to decay and leads to the brane configuration in Figure 2B where some of the flavor D4-branes which are not straight branes are approaching to the NS5-brane. One regards these brane configurations as particular states in the magnetic gauge theory with the gauge group (2.5) and superpotential (2.6). When the $NS5_{-\theta}$ -brane is replaced by N'_c coincident D6-branes, the brane configuration of Figure 2B is the same as the one studied in [6] where the gauge group was $SU(n_f - n_c)$ and the matter contents were n_f fundamentals and gauge singlet. Then the present number N'_c corresponds to n_f while N_c corresponds to n_c . This is equivalent to gauge the $U(n_f)$ global symmetry of [6] in the low energy.

At first, in order to obtain the supersymmetric vacua, one solves the F-term equations for the superpotential (2.6):

$$\begin{aligned} MY &= 0, & \tilde{Y}M &= 0, \\ -\frac{1}{\Lambda}Y\tilde{Y} &= m - \alpha M. \end{aligned} \tag{2.7}$$

The last relation, by multiplying M both sides, implies the following matrix equation $mM = \alpha M^2$. Since the eigenvalues for the meson field M are either 0 or $\frac{m}{\alpha}$, one takes $N'_c \times N'_c$ matrix

M with k 's eigenvalues 0 and $(N'_c - k)$'s eigenvalues $\frac{m}{\alpha}$ as follows:

$$M = \begin{pmatrix} 0 & 0 \\ 0 & \frac{m}{\alpha} \mathbf{1}_{N'_c - k} \end{pmatrix} \quad (2.8)$$

where $k = 1, 2, \dots, N'_c$ and $\mathbf{1}_{N'_c - k}$ is the $(N'_c - k) \times (N'_c - k)$ identity matrix [7]. In the brane configuration of Figure 2B, the k of the N'_c flavor D4-branes are connected with k of \tilde{N}_c color D4-branes and the resulting D4-branes stretch from the $NS5_{-\theta}$ -brane to the NS5-brane directly and the coordinate of an intersection point between the k D4-branes and the NS5-brane is given by $(v, w) = (+v_{NS5_{-\theta}}, 0)$. This corresponds to exactly the k 's eigenvalues 0 of M above (2.8). Now the remaining $(N'_c - k)$ flavor D4-branes between the $NS5_{-\theta}$ -branes and the $NS5'_R$ -brane are related to the corresponding remaining eigenvalues of M (2.8), i.e., $\frac{m}{\alpha} \mathbf{1}_{N'_c - k}$. The coordinate of an intersection point between the $(N'_c - k)$ D4-branes and the $NS5'_R$ -brane is given by $(v, w) = (0, +v_{NS5_{-\theta}} \cot \theta)$.

After we substitute (2.8) into the last equation of (2.7) gives rise to

$$Y\tilde{Y} = \begin{pmatrix} m\Lambda \mathbf{1}_k & 0 \\ 0 & 0 \end{pmatrix}. \quad (2.9)$$

Since the rank of the left hand side of this is at most \tilde{N}_c , one must have more stringent bound $k \leq \tilde{N}_c$. In the k -th vacuum the gauge symmetry is broken to $SU(\tilde{N}_c - k)$ and the supersymmetric vacuum drawn in Figure 2B with $k = 0$ has $Y = \tilde{Y} = 0$ and the gauge group $SU(\tilde{N}_c)$ is unbroken. The expectation value of M in this case is given by $M = \frac{m}{\alpha} \mathbf{1}_{N'_c} = m\Lambda \cot \theta \mathbf{1}_{N'_c}$.

So far, the ground states are supersymmetric. On the other hand, the theory has many nonsupersymmetric meta-stable ground states. For the IR free region, $N_c < N'_c < \frac{3}{2}N_c$ [1], the magnetic theory is the effective low energy description of the asymptotically free electric gauge theory. When we rescale the meson field as $M = h\Lambda\Phi$, then the Kahler potential for Φ is canonical and the magnetic “quarks” are canonical near the origin of field space. Then the magnetic superpotential can be written in terms of Φ

$$W_{dual} = h\Phi Y\tilde{Y} + \frac{h^2\mu_\phi}{2} \text{tr} \Phi^2 - h\mu^2 \text{tr} \Phi. \quad (2.10)$$

From this, one can read off the following quantities

$$\mu^2 = -m\Lambda, \quad \mu_\phi = -\alpha\Lambda^2, \quad M = h\Lambda\Phi.$$

The classical supersymmetric vacua given by (2.8) and (2.9) can be described as

$$h\Phi = \begin{pmatrix} 0 & 0 \\ 0 & \frac{\mu^2}{\mu_\phi} \mathbf{1}_{N'_c - k} \end{pmatrix}, \quad Y\tilde{Y} = \begin{pmatrix} \mu^2 \mathbf{1}_k & 0 \\ 0 & 0 \end{pmatrix}.$$

Now one splits, as in [6, 7], the $(N'_c - k) \times (N'_c - k)$ block at the lower right corner of $h\Phi$ and $Y\tilde{Y}$ into blocks of size n and $(N'_c - k - n)$ as follows:

$$h\Phi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & h\Phi_n & 0 \\ 0 & 0 & \frac{\mu^2}{\mu_\phi} \mathbf{1}_{N'_c - k - n} \end{pmatrix}, \quad Y\tilde{Y} = \begin{pmatrix} \mu^2 \mathbf{1}_k & 0 & 0 \\ 0 & \varphi \tilde{\varphi} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (2.11)$$

Here φ and $\tilde{\varphi}$ are $n \times (\tilde{N}_c - k)$ dimensional matrices and correspond to n flavors of fundamentals of the gauge group $SU(\tilde{N}_c - k)$ which is unbroken by the nonzero expectation value of Y and \tilde{Y} (2.9). In the brane configuration from Figure 2B, they correspond to fundamental strings connecting between the n flavor D4-branes and $(\tilde{N}_c - k)$ color D4-branes. Moreover, the Φ_n and $\varphi \tilde{\varphi}$ are $n \times n$ matrices. The supersymmetric ground state corresponds to the vacuum expectation values by $h\Phi_n = \frac{\mu^2}{\mu_\phi} \mathbf{1}_n$, $\varphi = 0 = \tilde{\varphi}$.

The full one loop potential for $\Phi_n, \varphi, \tilde{\varphi}$ takes the form in [6] and differentiating this potential with respect to Φ_n and putting $\varphi = 0 = \tilde{\varphi}$, one obtains

$$h\Phi_n \simeq \frac{\mu_\phi^*}{\tilde{N}_c} \mathbf{1}_n \quad \text{or} \quad M_n \simeq \frac{\alpha \Lambda^3}{\tilde{N}_c} \mathbf{1}_n \quad (2.12)$$

for real μ and we assume here that $\mu_\phi \ll \mu \ll \Lambda_m$. The vacuum energy V is given by $V \simeq n|h\mu^2|^2$ and expanding around this solution, one obtains the eigenvalues for mass matrix for φ and $\tilde{\varphi}$ and the vacuum (2.12) is locally stable.

The n flavor D4-branes of brane configuration in Figure 2B can bend due to the fact that there exists an attractive gravitational interaction between those flavor D4-branes and the NS5-brane from the DBI action, by following the procedure of [11]. The correct choice for the ground state of the system depends on the parameters y_1, y_2 and $v_{NS5-\theta}$.

One can move n D4-branes, from $(N'_c - k)$ D4-branes stretched between the $NS5'_R$ -brane and the $NS5_{-\theta}$ -brane at $w = +v_{NS5_{-\theta}} \cot \theta$, to the local minimum of the potential and the end points of these n D4-branes are at a nonzero w as in Figure 2B [6]. The remaining $(N'_c - k - n)$ flavor D4-branes between the $NS5_{-\theta}$ -brane and the $NS5'_R$ -brane are related to the corresponding eigenvalues of $h\Phi$ (2.11), i.e., $\frac{\mu^2}{\mu_\phi} \mathbf{1}_{N'_c - k - n}$. The coordinate of an intersection point between the $(N'_c - k - n)$ D4-branes and the $NS5'_R$ -brane is given by $(v, w) = (0, +v_{NS5_{-\theta}} \cot \theta)$. As we mentioned, the k D4-branes stretching from the $NS5_{-\theta}$ -brane to the NS5-brane correspond to exactly the k 's eigenvalues 0 of $h\Phi$ (2.11). Finally, the remnant n ‘‘curved’’ flavor D4-branes between the $NS5_{-\theta}$ -branes and the $NS5'_R$ -brane are related to the corresponding eigenvalues (2.12) of $h\Phi_n$. Since the eigenvalues of (2.12) are much smaller than $\frac{\mu^2}{\mu_\phi} \mathbf{1}_{N'_c - k - n}$ of (2.11), in Figure 2B, the n curved flavor D4-branes, instead of $(N'_c - k - n)$ flavor D4-branes, are nearer to the NS5-brane. By explicit computation as in [6], it can be shown that the local minimum occurs at $w \simeq \tan \theta \frac{y^4}{\ell_s^2 v_{NS5_{-\theta}}}$ with $x^6 \equiv y$.

Therefore, the classical brane construction can generalize the gauge theory discussion to the regime where the angle θ is of order one and different length parameters are of order ℓ_s or larger. Note that the gauge theory analysis is valid only when θ and $\frac{v_{NS5-\theta}}{\ell_s}$ are much smaller than ℓ_s [6].

3 Meta-stable brane configuration with three NS-branes plus O4-plane

3.1 Electric theory

The type IIA brane configuration [25] corresponding to $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group

$$Sp(N_c) \times SO(2N'_c) \quad (3.1)$$

and a bifundamental X that is in the representation $(\mathbf{2N}_c, \mathbf{2N}'_c)$ under the gauge group (3.1) can be described by a middle NS5-brane(012345), the left $NS5'_L$ -brane(012389) and the right $NS5'_R$ -brane(012389), $2N_c$ - and $2N'_c$ -color D4-branes(01236) as well as an O4-plane(01236). We take the arbitrary number of color D4-branes with the constraint $N'_c \geq N_c + 2$. The O4-plane acts as $(x^4, x^5, x^7, x^8, x^9) \rightarrow (-x^4, -x^5, -x^7, -x^8, -x^9)$ as usual. The bifundamental X corresponds to 4-4 strings connecting the $2N_c$ -color D4-branes with $2N'_c$ -color D4-branes.

The middle NS5-brane is located at $x^6 = 0$ and the x^6 coordinates for the $NS5'_L$ -brane and $NS5'_R$ -brane by $x^6 = -y_2$ and $x^6 = y_1$ respectively. The $2N_c$ D4-branes and $O4^+$ -plane are suspended between the middle NS5-brane and $NS5'_R$ -brane while the $2N'_c$ D4-branes and $O4^-$ -plane are suspended between the $NS5'_L$ -brane and the middle NS5-brane. We draw this brane configuration in Figure 3A [25] for the vanishing mass for the bifundamental X ⁴.

The gauge couplings of $Sp(N_c)$ and $SO(2N'_c)$ are given by

$$g_{Sp}^2 = \frac{g_s \ell_s}{y_1}, \quad g_{SO}^2 = \frac{g_s \ell_s}{y_2}$$

respectively. As y_2 goes to the infinity, the $SO(2N'_c)$ gauge group becomes a global symmetry and the theory leads to SQCD with the gauge group $Sp(N_c)$ and N'_c flavors(or $2N'_c$ fields) in the vector representation. The opposite limit $y_1 \rightarrow \infty$ leads to SQCD with the gauge group $SO(2N'_c)$ with $2N_c$ fields in the fundamental representation.

⁴This is equivalent to the reduced brane realization of Figure 1 in [26] if we remove D6-branes completely. See also the relevant works appeared in [27, 28, 29] for supersymmetric vacua and [26, 19, 20] for nonsupersymmetric vacua in the product gauge group between symplectic and orthogonal gauge groups.

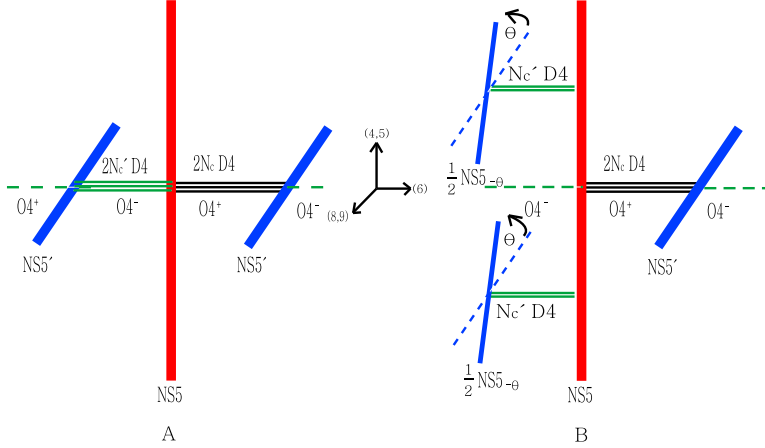


Figure 3: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $Sp(N_c) \times SO(2N'_c)$ and a bifundamental X with vanishing mass term(3A) and nonvanishing mass and quartic terms(3B) for the bifundamental. In Figure 3B, there are two deformations by rotation and displacement of $NS5'_L$ -brane and the superpotential is given by (3.2). Note that the mirrors for upper half $NS5_{-\theta}$ -brane and upper N'_c D4-branes in Figure 3B are preserved under the $O4$ -plane action.

There is no superpotential in Figure 3A. Let us deform this gauge theory. Displacing the two $NS5'$ -branes relative each other in the v direction corresponds to turning on a quadratic mass-deformed superpotential for the bifundamental X while rotating them in the w direction corresponds to turning on a quartic term for the bifundamental X . The deformed electric superpotential is as follows:

$$W_{elec} = -\frac{\alpha}{2} \text{tr}(XX)^2 + m \text{tr} XX, \quad \alpha = \frac{\tan \theta}{\Lambda}, \quad m = \frac{v_{NS5_{-\theta}}}{2\pi \ell_s^2} \quad (3.2)$$

where a symplectic metric(that has antisymmetric color indices) [26] is assumed in the $Sp(N_c)$ gauge group indices for a meson field XX . Half of $NS5_{-\theta}$ -brane with N'_c color D4-branes is moving to the $+v$ direction while the other half of $NS5_{-\theta}$ -brane with other N'_c color D4-branes is moving to $-v$ direction due to the $O4$ -plane for fixed $NS5'_R$ -brane. Then the x^5 coordinate of $NS5'_R$ -brane is zero and the x^5 coordinates of each half $NS5_{-\theta}$ -brane are given by $\pm v_{NS5_{-\theta}}$ respectively.

Giving an expectation value to the meson field XX corresponds to recombination of $2N_c$ - and $2N'_c$ - color D4-branes, which becomes $2N_c$ -color D4-branes, in Figure 3A such that they are suspended between the $NS5_{-\theta}$ -brane and the $NS5'_R$ -brane and pushing them into the w direction.

Now we draw this brane configuration in Figure 3B for nonvanishing mass for the bifundamental X by moving half of $NS5_{-\theta}$ -brane with N'_c color D4-branes to the $+v$ direction and

rotating it by an angle $-\theta$ in (w, v) -plane (and their mirrors). One can easily understand this brane configuration by adding O4-plane into the Figure 1B with appropriate number of color D4-branes. Compared with the brane configuration of [26, 25], the difference is the fact that there exists an extra rotation of $NS5_{-\theta}$ -brane. Of course, the $\theta = 0$ limit reduces to the one of [26, 25].

The solution for the supersymmetric vacua can be obtained by $XX = \frac{m}{\alpha}$ through the F-term condition for the superpotential (3.2). This breaks the gauge group $Sp(N_c) \times SO(2N'_c)$ to $Sp(N_c - k)$, $SO(2N'_c - 2k)$ and $U(2k)$. When the middle NS5-brane moves to $\pm w$ direction (half of them to $+w$ direction and half of them to $-w$ direction), then the three NS-branes ($NS5_{-\theta}$ -brane, $NS5'_R$ -brane and NS5-brane) intersect in three points in (v, w) -plane. In other words, the coordinates of (v, w) for those points are $(+v_{NS5_{-\theta}}, +v_{NS5_{-\theta}} \cot \theta)$, $(0, +v_{NS5_{-\theta}} \cot \theta)$ and $(0, +2v_{NS5_{-\theta}} \cot \theta)$. It is easy to see that the other intersection points are given by $(\pm v_{NS5_{-\theta}}, -v_{NS5_{-\theta}} \cot \theta)$, $(0, -v_{NS5_{-\theta}} \cot \theta)$, $(0, 0)$, $(-v_{NS5_{-\theta}}, +v_{NS5_{-\theta}} \cot \theta)$ and $(0, -2v_{NS5_{-\theta}} \cot \theta)$. Then $2(N_c - k)$ D4-branes are connecting between the middle NS5-brane and the $NS5'_R$ -brane. The $2(N'_c - k)$ D4-branes are connecting between the $NS5_{-\theta}$ -brane and the middle NS5-brane. Finally, $2k$ D4-branes are connecting between the $NS5_{-\theta}$ -brane and the $NS5'_R$ -brane directly. The distance from $2k$ D4-branes to the middle NS5-brane can be read off from the trigonometric geometry and the w coordinate is given by $w = \pm v_{NS5_{-\theta}} \cot \theta$.

3.2 Magnetic theory

By applying the Seiberg dual to the $Sp(N_c)$ factor in (3.1), starting from Figure 3B and moving the NS5-brane to the right all the way past the $NS5'_R$ -brane, one obtains the Figure 4A. Before arriving at the Figure 4A, there exists an intermediate step where the N'_c D4-branes are connecting between half $NS5_{-\theta}$ -brane and $NS5'_R$ -brane (and their mirrors) and $2\tilde{N}_c$ D4-branes connecting between $NS5'_R$ -brane and NS5-brane. By introducing $2N'_c$ D4-branes and $2N'_c$ anti-D4-branes between $NS5'_R$ -brane and NS5-brane, recombining half of the former with the N'_c D4-branes that are connecting between half $NS5_{-\theta}$ -brane and $NS5'_R$ -brane and moving those combined D4-branes to $+v$ -direction (and their mirrors), one gets the final Figure 4A where we are left with $2(N'_c - \tilde{N}_c)$ anti-D4-branes between $NS5'_R$ -brane and NS5-brane.

Then the gauge group is given by

$$Sp(\tilde{N}_c = N'_c - N_c - 2) \times SO(2N'_c) \quad (3.3)$$

where the number of dual color was obtained from the linking number counting, as done in [26, 25]. The matter contents are the field Y in the bifundamental representation $(\mathbf{2}\tilde{\mathbf{N}}_c, \mathbf{2N}'_c)$

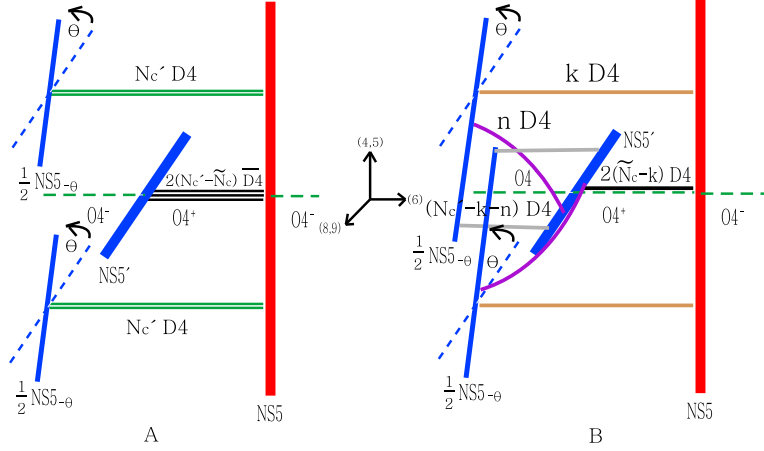


Figure 4: The magnetic brane configuration corresponding to Figure 3B with D4- and $\overline{D4}$ -branes(4A) when the distance between $NS5_{-\theta}$ -brane and the $NS5'_R$ -brane along v direction is large and with a misalignment between D4-branes(4B) when they are close to each other. The upper N'_c flavor D4-branes connecting between the upper half $NS5_{-\theta}$ -brane and $NS5'_R$ -brane are splitting into $(N'_c - k)$ and k D4-branes. The location of intersection between the upper half $NS5_{-\theta}$ -brane and the upper $(N'_c - k)$ D4-branes is given by $(v, w) = (0, +v_{NS5_{-\theta}} \cot \theta)$ while the one between the upper half $NS5_{-\theta}$ -brane and the upper k D4-branes is given by $(v, w) = (+v_{NS5_{-\theta}}, 0)$. By moving n flavor D4-branes from the upper $(N'_c - k)$ flavor D4-branes, the nonzero positive w coordinate for n “curved” flavor D4-branes is determined later. Similarly, the location of intersection between the lower half $NS5_{-\theta}$ -brane and the lower $(N'_c - k)$ D4-branes is given by $(v, w) = (0, -v_{NS5_{-\theta}} \cot \theta)$ while the one between the lower half $NS5_{-\theta}$ -brane and the lower k D4-branes is given by $(v, w) = (-v_{NS5_{-\theta}}, 0)$.

under the dual gauge group (3.3) and the gauge-singlet $M(\equiv XX)$ is in the adjoint representation for the second dual gauge group $(\mathbf{1}, \mathbf{N}'_c(2\mathbf{N}'_c - \mathbf{1}))$ under the dual gauge group (3.3). The quantum corrections can be understood for small $v_{NS5_{-\theta}}$ by using the low energy field theory on the branes. The low energy dynamics of the magnetic brane configuration can be described by the $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group (3.3) and the gauge couplings for the two gauge group factors are given by

$$g_{Sp, mag}^2 = \frac{g_s \ell_s}{y_1}, \quad g_{SO, mag}^2 = \frac{g_s \ell_s}{(y_2 - y_1)}.$$

Then the dual magnetic superpotential, by adding the mass term (3.2) for the bifundamental X , which can be interpreted as a linear term in the meson M , and quartic term to the cubic superpotential, is given by

$$W_{dual} = \frac{1}{\Lambda} MYY - \frac{\alpha}{2} M^2 + mM. \quad (3.4)$$

Of course, the brane configuration for zero mass for the bifundamental can be obtained from Figure 4A by recombination between half $NS5_{-\theta}$ -branes together with color D4-branes via

pushing them into the origin $v = 0$. Then the number of dual colors for D4-branes becomes $2N'_c$ between the $NS5_{-\theta}$ -brane and the $NS5'_R$ -brane and $2\tilde{N}_c$ between $NS5'_R$ -brane and NS5-brane. Moreover, the zero limit of quartic term for the bifundamental can be done by taking $\theta \rightarrow 0$ for the $NS5_{-\theta}$ -branes.

The conditions $b_{Sp(\tilde{N}_c)}^{mag} = 4N'_c - 6N_c - 6 < 0$ and $b_{Sp(N_c)} = 3(2N_c + 2) - 2N'_c > 0$ lead to $N'_c < \frac{3}{2}N_c + \frac{3}{2}$. The range for the N'_c can be written as $N_c + 2 < N'_c < \frac{3}{2}N_c + \frac{3}{2}$. Moreover, $b_{SO(2N'_c)} = 3(2N'_c - 2) - 2N_c > 0$ and $b_{SO(2N'_c)}^{mag} = 2(N_c + N'_c) > 0$. At the scale Λ_1 , the $Sp(N_c)$ theory is strongly coupled and the Seiberg duality occurs. Then at energy scale lower than Λ_1 , the theory is weakly coupled. One cannot ignore the contributions from the coupling of $SO(2N'_c)^{mag}$. Then under the constraint, $\Lambda_2 \ll \left(\frac{\Lambda_1}{\mu}\right)^b \Lambda_1 \ll \Lambda_1$ where $b \equiv \frac{b_{Sp(\tilde{N}_c)}^{mag} - b_{SO(2N'_c)}^{mag}}{b_{SO(2N'_c)}}$, one can ignore the contribution from the gauge coupling of $SO(2N'_c)^{mag}$ at the supersymmetry breaking scale.

The brane configuration in Figure 4A is stable as long as the distance $v_{NS5_{-\theta}}$ between the upper half $NS5_{-\theta}$ -brane and $NS5'_R$ -brane is large, as in [11, 25]. If they are close to each other, then this brane configuration is unstable to decay to the brane configuration in Figure 4B with bending effect of tilted D4-branes connecting half $NS5_{-\theta}$ -brane and $NS5'_R$ -brane. Of course, this brane realization can be obtained from the Figure 2B by adding O4-plane with appropriate mirrors. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group (3.3) and superpotential (3.4). When the two half $NS5_{-\theta}$ -branes are replaced by the coincident $2N'_c$ D6-branes, the brane configuration of Figure 4B is the same as the one studied in [6] together with an addition of appropriate O4-plane.

In order to obtain the supersymmetric vacua, one solves the F-term equations for the superpotential (3.4):

$$MY = 0, \quad -\frac{1}{\Lambda}YY = m - \alpha M. \quad (3.5)$$

The matrix equation $mM = \alpha M^2$ implies that the eigenvalues for the meson field M are either 0 or $\frac{m}{\alpha}$, one takes $2N'_c \times 2N'_c$ matrix with $2k$'s eigenvalues 0 and $2(N'_c - k)$'s eigenvalues $\frac{m}{\alpha}$:

$$M = \begin{pmatrix} 0 & 0 \\ 0 & \frac{m}{\alpha} \mathbf{1}_{N'_c - k} \otimes i\sigma_2 \end{pmatrix} \quad (3.6)$$

where $k = 1, 2, \dots, 2N'_c$ and $\mathbf{1}_{N'_c - k}$ is the $(N'_c - k) \times (N'_c - k)$ identity matrix [7]⁵. Therefore, in the brane configuration of Figure 4B, the k of the upper N'_c flavor D4-branes are connected

⁵The mass matrix m is antisymmetric in the indices and is given by $m = \text{diag}(i\sigma_2 m_1, i\sigma_2 m_2, \dots, i\sigma_2 m_{N'_c})$ due to the antisymmetric matrix M . In the matrix equation mM , we assumed this property of mass matrix. In (3.6), we used the equal mass as $m \equiv m_1 = m_2 = \dots = m_{N'_c}$ unfortunately.

with k of \tilde{N}_c color D4-branes and the resulting D4-branes stretch from the upper $NS5_{-\theta}$ -brane to the NS5-brane directly and the coordinate of an intersection point between the k upper D4-branes and the NS5-brane is given by $(v, w) = (+v_{NS5_{-\theta}}, 0)$. Similarly the mirrors are located at $(v, w) = (-v_{NS5_{-\theta}}, 0)$. This corresponds to exactly the k 's eigenvalues 0 of M above (3.6). Now the remaining $(N'_c - k)$ upper flavor D4-branes between the $NS5_{-\theta}$ -branes and the $NS5'_R$ -brane are related to the corresponding half eigenvalues of M which is equal to $\frac{m}{\alpha} \mathbf{1}_{N'_c - k} \otimes i\sigma_2$. The coordinate of an intersection point between the $(N'_c - k)$ upper D4-branes and the $NS5'_R$ -brane is given by $(v, w) = (0, +v_{NS5_{-\theta}} \cot \theta)$ corresponding to positive eigenvalues of M . The mirrors are located at $(v, w) = (0, -v_{NS5_{-\theta}} \cot \theta)$ corresponding to negative eigenvalues of M .

After we substitute (3.6) into the second equation of (3.5) gives rise to

$$YY = \begin{pmatrix} m\Lambda \mathbf{1}_{2k} & 0 \\ 0 & 0 \end{pmatrix}. \quad (3.7)$$

Since the rank of the left hand side of this is at most $2\tilde{N}_c$, one must have more stringent bound $k \leq 2\tilde{N}_c$. In the k -th vacuum the gauge symmetry is broken to $Sp(\tilde{N}_c - k)$ and the supersymmetric vacuum drawn in Figure 4B with $k = 0$ has $Y = 0$ and the gauge group $Sp(\tilde{N}_c)$ is unbroken. The expectation value of M (3.6) in this case is given by $M = \frac{m}{\alpha} \mathbf{1}_{N'_c} \otimes i\sigma_2 = m\Lambda \cot \theta \mathbf{1}_{N'_c} \otimes i\sigma_2$.

The theory has many nonsupersymmetric meta-stable ground states. For the IR free region, $N_c + 2 < N'_c < \frac{3}{2}(N_c + 1)$ [1, 13], the magnetic theory is the effective low energy description of the asymptotically free electric gauge theory. When we rescale the meson field as $M = h\Lambda\Phi$, then the Kahler potential for Φ is canonical and the magnetic “quarks” are canonical near the origin of field space. Then the magnetic superpotential can be written in terms of Φ

$$W_{dual} = h\Phi YY + \frac{h^2\mu_\phi}{2} \text{tr } \Phi^2 - h\mu^2 \text{tr } \Phi.$$

The classical supersymmetric vacua given by (3.6) and (3.7) can be described as

$$h\Phi = \begin{pmatrix} 0 & 0 \\ 0 & \frac{\mu^2}{\mu_\phi} \mathbf{1}_{N'_c - k} \otimes i\sigma_2 \end{pmatrix}, \quad YY = \begin{pmatrix} \mu^2 \mathbf{1}_{2k} & 0 \\ 0 & 0 \end{pmatrix}.$$

Now one splits, as in [6, 7], the $2(N'_c - k) \times 2(N'_c - k)$ block at the lower right corner of $h\Phi$ and YY into blocks of size $2n$ and $2(N'_c - k - n)$ as follows:

$$h\Phi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & h\Phi_{2n} & 0 \\ 0 & 0 & \frac{\mu^2}{\mu_\phi} \mathbf{1}_{N'_c - k - n} \otimes i\sigma_2 \end{pmatrix}, \quad YY = \begin{pmatrix} \mu^2 \mathbf{1}_{2k} & 0 & 0 \\ 0 & \varphi\varphi & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (3.8)$$

Here φ is $2n \times 2(\tilde{N}_c - k)$ dimensional matrix and corresponds to $2n$ flavors of fundamentals of the gauge group $Sp(\tilde{N}_c - k)$ which is unbroken by the nonzero expectation value of Y . In the brane configuration in Figure 4B, they correspond to fundamental strings connecting the n upper flavor D4-branes and $(\tilde{N}_c - k)$ color D4-branes (and their mirrors). The Φ_{2n} and $\varphi\varphi$ are $2n \times 2n$ matrices. The supersymmetric ground state corresponds to the vacuum expectation values by $h\Phi_{2n} = \frac{\mu^2}{\mu_\phi} \mathbf{1}_n \otimes i\sigma_2, \varphi = 0$.

The full one loop potential for Φ_{2n}, φ takes the similar form in [6] and differentiating this potential with respect to Φ_{2n} and putting $\varphi = 0$, one obtains

$$h\Phi_{2n} \simeq \frac{\mu_\phi^*}{\tilde{N}_c} \mathbf{1}_n \otimes i\sigma_2 \quad \text{or} \quad M_{2n} \simeq \frac{\alpha\Lambda^3}{\tilde{N}_c} \mathbf{1}_n \otimes i\sigma_2 \quad (3.9)$$

for real μ and we assume here that $\mu_\phi \ll \mu \ll \Lambda_m$. The vacuum energy V is given by $V \simeq n|h\mu^2|^2$ and expanding around this solution, one obtains the eigenvalues for mass matrix for φ and the vacuum (3.9) is locally stable.

The n flavor D4-branes of straight brane configuration of Figure 4B can bend due to the fact that there exists an attractive gravitational interaction between those flavor D4-branes and the NS5-brane from the DBI action, by following the procedure of [11, 25]. The correct choice for the ground state of the system depends on the parameters y_1, y_2 and $v_{NS5-\theta}$.

One can move n upper D4-branes, from upper $(N'_c - k)$ D4-branes stretched between the $NS5'_R$ -brane and the upper $NS5_{-\theta}$ -brane at $w = +v_{NS5_{-\theta}} \cot \theta$, to the local minimum of the potential and the end points of these n D4-branes are at a nonzero w as in Figure 4B. The remaining upper $(N'_c - k - n)$ flavor D4-branes between the upper $NS5_{-\theta}$ -brane and the $NS5'_R$ -brane are related to the corresponding “positive” eigenvalues of $h\Phi$ (3.8) which is equal to $\frac{\mu^2}{\mu_\phi} \mathbf{1}_{(N'_c - k - n)} \otimes i\sigma_2$. The coordinate of an intersection point between the upper $(N'_c - k - n)$ D4-branes and the $NS5'_R$ -brane is given by $(v, w) = (0, +v_{NS5_{-\theta}} \cot \theta)$. The remnant n upper “curved” flavor D4-branes between the $NS5_{-\theta}$ -branes and the $NS5'_R$ -brane are related to the corresponding “positive” eigenvalues (3.9) of $h\Phi_{2n}$. As we mentioned, the k D4-branes stretching from the $NS5_{-\theta}$ -brane to the NS5-brane correspond to exactly the k ’s eigenvalues 0 of $h\Phi$ (3.8). By explicit computation it can be shown that the local minimum occurs at $w \simeq \tan \theta \frac{y^4}{\ell_s^2 v_{NS5_{-\theta}}}$ with $x^6 \equiv y$.

Note that the intersection point between the lower $(N'_c - k)$ D4-branes and the lower half $NS5_{-\theta}$ -brane is located at $w = -v_{NS5_{-\theta}} \cot \theta$. The lower $(N'_c - k - n)$ flavor D4-branes are related to the corresponding “negative” eigenvalues of $h\Phi$ (3.8).

3.3 Other magnetic theory

By applying the Seiberg dual to the $SO(2N'_c)$ factor in (3.1), starting from modified Figure 3B, where the x^5 coordinate of $NS5'_L$ -brane is equal to zero and the x^5 coordinates of half $NS5_{-\theta}$ -branes which were $NS5'_R$ -brane are $\pm v_{NS5_{-\theta}}$, and moving the NS5-brane to the left all the way past the $NS5'_L$ -brane, one obtains the magnetic brane configuration similar to Figure 4A. The gauge group is given by

$$Sp(N_c) \times SO(2\tilde{N}'_c = 2N_c - 2N'_c + 4). \quad (3.10)$$

The matter contents are the field Y in the bifundamental representation $(\mathbf{2N}_c, \mathbf{2\tilde{N}'_c})$ under the dual gauge group (3.10) and the gauge-singlet M is in the adjoint representation for the first dual gauge group, i.e., a symmetric matrix, $(\mathbf{N}_c(\mathbf{2N}_c + \mathbf{1}), \mathbf{1})$ under the dual gauge group. The superpotential is the same as the one in (3.4) and the corresponding modified Figure 4B, which is exactly a reflection of Figure 4B with respect to the NS5-brane, i.e., all the D4-branes, $NS5_{-\theta}$ -brane and $NS5'_R$ -brane are located at the right hand side of NS5-brane, can be constructed similarly. The discussion for the supersymmetric vacua in previous subsection can be applied here also ⁶.

The theory has many nonsupersymmetric meta-stable ground states. For the IR free region, $2N'_c - 4 < 2N_c < \frac{3}{2}(2N'_c - 2)$ [1, 13], the magnetic theory is the effective low energy description of the asymptotically free electric gauge theory. When we rescale the meson field as $M = h\Lambda\Phi$, then the Kahler potential for Φ is canonical and the magnetic “quarks” are canonical near the origin of field space. Then the magnetic superpotential can be written in terms of Φ

$$W_{dual} = h\Phi YY + \frac{h^2\mu_\phi}{2} \text{tr } \Phi^2 - h\mu^2 \text{tr } \Phi.$$

The classical supersymmetric vacua can be described as

$$h\Phi = \begin{pmatrix} 0 & 0 \\ 0 & \frac{\mu^2}{\mu_\phi} \mathbf{1}_{N_c-k} \otimes \sigma_3 \end{pmatrix}, \quad YY = \begin{pmatrix} \mu^2 \mathbf{1}_{2k} & 0 \\ 0 & 0 \end{pmatrix}.$$

Now one splits, as in [6, 7], the $2(N_c - k) \times 2(N_c - k)$ block at the lower right corner of $h\Phi$ and YY into blocks of size $2n$ and $2(N_c - k - n)$ as follows:

$$h\Phi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & h\Phi_{2n} & 0 \\ 0 & 0 & \frac{\mu^2}{\mu_\phi} \mathbf{1}_{N_c-k-n} \otimes \sigma_3 \end{pmatrix}, \quad YY = \begin{pmatrix} \mu^2 \mathbf{1}_{2k} & 0 & 0 \\ 0 & \varphi\varphi & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

⁶In this case, the mass matrix m is symmetric in the indices and is given by $m = \text{diag}(\sigma_3 m_1, \sigma_3 m_2, \dots, \sigma_3 m_{N_c})$ due to the symmetric matrix M . In the matrix equation mM , we assumed this property of mass matrix.

Here φ is $2n \times 2(\tilde{N}'_c - k)$ dimensional matrix and corresponds to $2n$ flavors of fundamentals of the gauge group $SO(2\tilde{N}'_c - 2k)$ which is unbroken by the nonzero expectation value of Y . In the brane configuration, they correspond to fundamental strings connecting the n upper flavor D4-branes and $(\tilde{N}'_c - k)$ color D4-branes (and their mirrors). The Φ_{2n} and $\varphi\varphi$ are $2n \times 2n$ matrices. The supersymmetric ground state corresponds to $h\Phi_{2n} = \frac{\mu^2}{\mu_\phi} \mathbf{1}_n \otimes \sigma_3, \varphi = 0$.

The full one loop potential for Φ_{2n}, φ takes the similar form in [6] and differentiating this potential with respect to Φ_{2n} and putting $\varphi = 0$, one obtains

$$h\Phi_{2n} \simeq \frac{\mu_\phi^*}{\tilde{N}'_c} \mathbf{1}_n \otimes \sigma_3 \quad \text{or} \quad M_{2n} \simeq \frac{\alpha\Lambda^3}{\tilde{N}'_c} \mathbf{1}_n \otimes \sigma_3 \quad (3.11)$$

for real μ and we assume here that $\mu_\phi \ll \mu \ll \Lambda_m$. The vacuum energy V is given by $V \simeq n|h\mu^2|^2$ and expanding around this solution, one obtains the eigenvalues for mass matrix for φ and the vacuum (3.11) is locally stable. One can also analyze the correspondence between the eigenvalues of $h\Phi$ and the w coordinates for the flavor D4-branes.

4 Meta-stable brane configuration with five NS-branes plus O6-plane

4.1 Electric theory

The type IIA brane configuration [25] corresponding to $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group

$$SU(N_c) \times SU(N'_c) \quad (4.1)$$

and the symmetric flavor for $SU(N_c)$, the conjugate symmetric flavor for $SU(N_c)$, a bifundamental X in the representation $(\mathbf{N}_c, \overline{\mathbf{N}}'_c)$ and its conjugate field \tilde{X} in the representation $(\overline{\mathbf{N}}_c, \mathbf{N}'_c)$, under the gauge group can be described similarly. It consists of a middle NS5-brane(012345), the left $NS5_L$ -brane(012345) and the right $NS5_R$ -brane(012345), the left $NS5'_L$ -brane(012389) and the right $NS5'_R$ -brane(012389), N_c - and N'_c -color D4-branes(01236) and an $O6^+$ -plane(0123789). We take the arbitrary number of color D4-branes with the constraint $2N'_c \geq N_c$. The $O6^+$ -plane acts as $(x^4, x^5, x^6) \rightarrow (-x^4, -x^5, -x^6)$ and has RR charge +4 playing the role of +4 D6-branes. The bifundamentals X and \tilde{X} correspond to 4-4 strings connecting the N_c -color D4-branes with N'_c -color D4-branes while the symmetric and conjugate symmetric flavors correspond to 4-4 strings connecting N_c D4-branes located at negative x^6 region and N_c D4-branes located at positive x^6 region ⁷.

⁷ See also the relevant works in [16, 14, 17, 30] for supersymmetric vacua and [31, 32, 33, 34, 20] for nonsupersymmetric vacua in the presence of O6-plane.

The middle NS5-brane is located at $x^6 = 0$ and the x^6 coordinates for the $NS5_L$ -brane, $NS5'_L$ -brane, $NS5'_R$ -brane and $NS5_R$ -brane are given by $x^6 = -y_2, -y_1, y_1$ and $x^6 = y_2$ respectively. The N_c D4-branes are suspended between the $NS5'_L$ -brane (whose x^6 coordinate is given by $x^6 = -y_1$) and $NS5'_R$ -brane (whose x^6 coordinate is given by $x^6 = y_1$) while the N'_c D4-branes are suspended between the $NS5_L$ -brane and the $NS5'_L$ -brane (and moreover they are suspended between the $NS5'_R$ -brane and the $NS5_R$ -brane). We draw this brane configuration in Figure 5A [25] for the vanishing mass for the bifundamentals⁸.

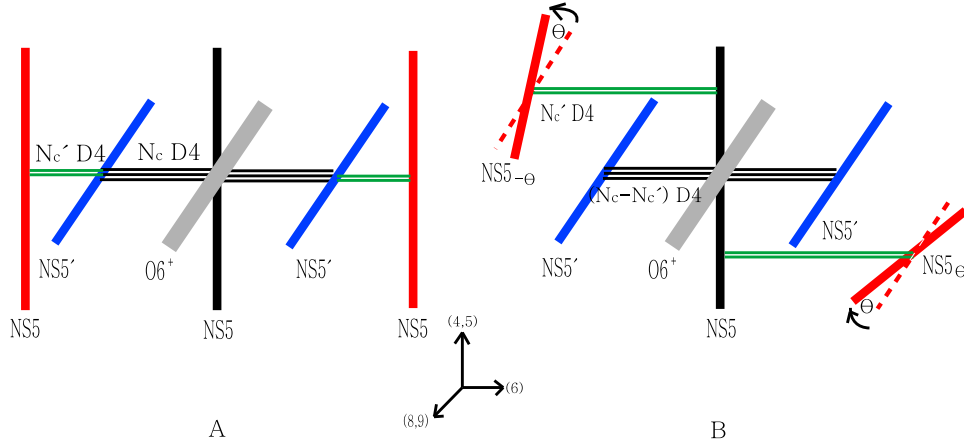


Figure 5: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $SU(N_c) \times SU(N'_c)$ and the bifundamentals X and \tilde{X} as well as symmetric and conjugate symmetric flavors with vanishing mass term (5A) and nonvanishing mass and quartic terms (5B) for the bifundamentals. In Figure 5B, in addition to the mass deformation, the deformation by the rotation of $NS5_L$ -brane by an angle $-(\frac{\pi}{2} - \theta)$ in (w, v) -plane arises. The superpotential of Figure 5B is characterized by (4.3) and the mirrors are preserved under the O6-plane action.

The gauge couplings of $SU(N_c)$ and $SU(N'_c)$ are given by

$$g_1^2 = \frac{g_s \ell_s}{y_1}, \quad g_2^2 = \frac{g_s \ell_s}{y_2}. \quad (4.2)$$

As y_2 goes to ∞ , the $SU(N'_c)$ gauge group becomes a global symmetry and the theory leads to SQCD-like theory with the gauge group $SU(N_c)$ with symmetric and conjugate symmetric flavors and N'_c fundamental flavors.

According to result of [31], there is no electric superpotential corresponding to the Figure 5A. Now let us deform this theory⁹. Displacing the two NS5-branes relative each other in the v direction corresponds to turning on a quadratic superpotential for the bifundamentals

⁸This is equivalent to the reduced brane configuration of Figure 1 in [31] with particular rotations for the NS-branes if we ignore all the D6-branes completely.

⁹Another deformation corresponds to a rotation θ' of the $NS5'_L$ -brane and the $NS5'_R$ -brane in (w, v) -plane.

X and \tilde{X} and further rotation of $NS5_L$ -brane by an angle $-(\frac{\pi}{2} - \theta)$ in (w, v) -plane (and $NS5_R$ -brane by an angle $+(\frac{\pi}{2} - \theta)$ in (w, v) -plane) provides the following deformed electric superpotential

$$W_{elec} = -\frac{\alpha}{2} \text{tr}(X\tilde{X})^2 + m \text{tr} X\tilde{X}, \quad \alpha = \frac{\tan \theta}{\Lambda}, \quad m = \frac{v_{NS5-\theta}}{2\pi\ell_s^2}. \quad (4.3)$$

The $NS5_{-\theta}$ -brane is moving to the $+v$ direction together with N'_c D4-branes while the $NS5_{\theta}$ -brane is moving to $-v$ direction due to the O6-plane for fixed NS5-brane and $NS5'_{L,R}$ -branes.

We draw this brane configuration in Figure 5B for nonvanishing mass for the bifundamentals by moving the $NS5_L$ -brane with N'_c color D4-branes to the $+v$ direction (and their mirrors to $-v$ direction) and rotating it by an angle $-(\frac{\pi}{2} - \theta)$ in (w, v) -plane. Here we decompose N_c D4-branes connecting $NS5'_L$ -brane and $NS5'_R$ -brane into $(N_c - N'_c)$ D4-branes and N'_c D4-branes. Then the latter can move $+v$ direction together with same number of D4-branes connecting between the $NS5_{-\theta}$ -brane and the $NS5'_L$ -brane (and their mirrors) while the former are connecting between the $NS5'_L$ -brane and the NS5-brane (and their mirrors). Compared with the brane configuration of [25], the rotation of $NS5_L$ -brane is the only difference and $\theta \rightarrow \frac{\pi}{2}$ limit reduces to the one of [25].

The solution for the supersymmetric vacua can be obtained by $X\tilde{X} = \frac{m}{\alpha}$ through the F-term conditions. This breaks the gauge group $SU(N_c) \times SU(N'_c)$ to $SU(N_c - N'_c - k)$, $SU(N'_c - k)$, $SO(2k)$ and $U(k)$. When the middle NS5-brane moves to $+w$ direction, then the three NS-branes intersect in three points in (v, w) -plane. In other words, the coordinates of (v, w) for those points are $(v_{NS5_{-\theta}}, v_{NS5_{-\theta}} \cot \theta)$, $(0, v_{NS5_{-\theta}} \cot \theta)$ and $(0, 2v_{NS5_{-\theta}} \cot \theta)$. It is easy to see that there exists the other intersection point given by $(-v_{NS5_{-\theta}}, v_{NS5_{-\theta}} \cot \theta)$. Note that the distance along v coordinate for $NS5_{\theta}$ -brane is equal to the one for $NS5_{-\theta}$ -brane: $v_{NS5_{\theta}} = v_{NS5_{-\theta}}$. Then $(N_c - N'_c - k)$ D4-branes are connecting between the $NS5'_L$ -brane and the $NS5'_R$ -brane including the middle NS5-brane. The $(N'_c - k)$ D4-branes are connecting between the $NS5_{-\theta}$ -brane and the middle NS5-brane (and their mirrors). The k D4-branes are connecting between the $NS5_{-\theta}$ -brane and the $NS5'_L$ -brane (and their mirrors). Finally, $2k$ D4-branes are connecting between the $NS5'_L$ -brane and the $NS5'_R$ -brane directly without touching the middle NS5-brane. The distance from $2k$ D4-branes to the middle NS5-brane can be read off from the trigonometric geometry and it is given by $w = +v_{NS5_{-\theta}} \cot \theta$.

This introduces the dynamics of an adjoint of gauge group $SU(N_c)$ whose mass goes to infinity in the limit where the rotation angle goes to zero but is finite for generic nonzero rotation angle θ' . This field couples to the symmetric tensor S , its conjugate field \tilde{S} and bifundamentals X, \tilde{X} . Integrating it out leads to a further contribution to the quartic superpotential for bifundamentals (4.3). Moreover, there are also two terms $(S\tilde{S})^2$ and $S\tilde{S}X\tilde{X}$ with angle-dependent coefficient in the superpotential. These extra terms appear in the magnetic superpotential also. This is beyond the scope of the present paper.

4.2 Magnetic theory

Starting from the Figure 5B, we apply the Seiberg dual to the $SU(N_c)$ factor in (4.1), the $NS5'_L$ -brane and the $NS5'_R$ -brane are interchanged each other. Then the number of color \tilde{N}_c was given by $\tilde{N}_c = 2N'_c - N_c$ from [31, 22]. Then the N'_c D4-branes are connecting between the $NS5_{-\theta}$ -brane and the $NS5'_R$ -brane (and their mirrors) and \tilde{N}_c D4-branes are connecting between $NS5'_R$ -brane and the middle NS5-brane. By introducing N'_c D4-branes and N'_c anti-D4-branes between the $NS5'_R$ -brane and the NS5-brane, reconnecting the former with the N'_c D4-branes connecting between the $NS5_{-\theta}$ -brane and the $NS5'_R$ -brane and moving those combined D4-branes to $+v$ -direction (and their mirrors to $-v$ direction), one gets the final Figure 6A where we are left with $(N'_c - \tilde{N}_c)$ anti-D4-branes between the $NS5'_R$ -brane and the NS5-brane.

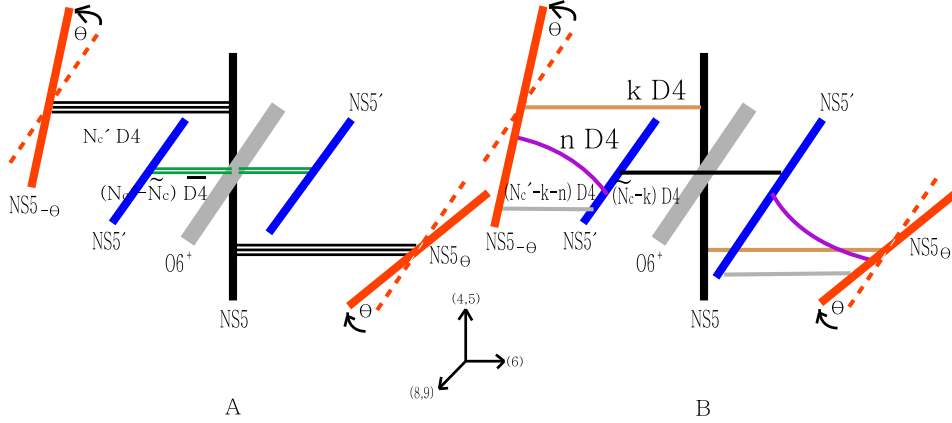


Figure 6: The magnetic brane configuration corresponding to Figure 5B with D4- and $\overline{D4}$ -branes (6A) when the distance between $NS5_{-\theta}$ -brane and the $NS5'_R$ -brane along v direction is large and with a misalignment between D4-branes (6B) when they are close to each other. The upper N'_c flavor D4-branes connecting between the $NS5_{-\theta}$ -brane and $NS5'_R$ -brane are splitting into $(N'_c - k)$ and k D4-branes. The location of intersection between the $NS5_{-\theta}$ -brane and the upper $(N'_c - k)$ D4-branes is given by $(v, w) = (0, +v_{NS5_{-\theta}} \cot \theta)$ while the one between the $NS5_{-\theta}$ -brane and the upper k D4-branes is given by $(v, w) = (+v_{NS5_{-\theta}}, 0)$. By moving n flavor D4-branes from the upper $(N'_c - k)$ flavor D4-branes, the nonzero positive w coordinate for n “curved” flavor D4-branes can be determined. Similarly, the location of intersection between the $NS5_{\theta}$ -brane and the lower $(N'_c - k)$ D4-branes is given by $(v, w) = (0, +v_{NS5_{\theta}} \cot \theta)$ while the one between the $NS5_{\theta}$ -brane and the lower k D4-branes is given by $(v, w) = (-v_{NS5_{\theta}}, 0)$ due to the O6-plane action.

The gauge group is given by

$$SU(\tilde{N}_c = 2N'_c - N_c) \times SU(N'_c) \quad (4.4)$$

where the number of dual color can be obtained from the linking number counting, as done in [31, 22]. The matter contents are the field Y in the bifundamental representation $(\tilde{\mathbf{N}}_c, \overline{\mathbf{N}}'_c)$ and its complex conjugate field \tilde{Y} in the bifundamental representation $(\overline{\tilde{\mathbf{N}}}_c, \mathbf{N}'_c)$, under the dual gauge group (4.4) and the gauge singlet $M \equiv X\tilde{X}$ in the representation for $(\mathbf{1}, \mathbf{N}'_c{}^2 - \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1})$ under the dual gauge group. There are also the symmetric flavor for $SU(\tilde{N}_c)$ and the conjugate symmetric flavor for $SU(\tilde{N}_c)$. A cubic superpotential is an interaction between dual “quarks” and a meson.

Then the dual magnetic superpotential, by adding the mass term like as (4.3) and the quartic term for the bifundamentals X and \tilde{X} to this cubic superpotential, is given by

$$W_{dual} = \frac{1}{\Lambda} M Y \tilde{Y} - \frac{\alpha}{2} M^2 + m M. \quad (4.5)$$

The brane configuration for zero mass for the bifundamentals can be obtained from Figure 6A by pushing the $NS5_{-\theta}$ -brane together with N'_c D4-branes into the origin $v = 0$. Then the number of dual colors for D4-branes becomes N'_c between the $NS5_{-\theta}$ -brane and the $NS5'_R$ -brane and \tilde{N}_c between the $NS5'_R$ -brane and the NS5-brane (and their mirrors). Note that there is no interaction term between the symmetric or conjugate symmetric flavors with other matter contents. During the dual process, the outer $NS5_{\pm\theta}$ -branes do not cross the middle NS5-brane.

The conditions $b_{SU(\tilde{N}_c)}^{mag} < 0$ and $b_{SU(N_c)} > 0$ imply that $N'_c < \frac{2}{3}N_c + \frac{2}{3}$. Then the range for the N'_c can be written as $\frac{1}{2}N_c < N'_c < \frac{2}{3}N_c + \frac{2}{3}$. Moreover, $b_{SU(N'_c)} = 3N'_c - N_c > 0$ and $b_{SU(N'_c)}^{mag} = N_c > 0$. Then one can also analyze the hierarchy of scales as before.

The brane configuration in Figure 6A is stable as long as the distance $v_{NS5_{-\theta}}$ between the upper $NS5_{-\theta}$ -brane and the $NS5'_R$ -brane is large, as in [11, 25]. If they are close to each other, then this brane configuration is unstable to decay and leads to the brane configuration in Figure 6B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group (4.4) and superpotential (4.5). When the $NS5_{-\theta}$ -brane is replaced by coincident N'_c D6-branes, the brane configuration of Figure 6B is the same as the Figure 3 studied in [8]. This brane configuration also can be obtained from the Figure 2B by adding O6-plane with appropriate mirrors.

One can solve the F-term equations (2.7) and one takes $N'_c \times N'_c$ matrix in (2.8). In the brane configuration of Figure 6B, the k of the N'_c flavor D4-branes are connecting with k of \tilde{N}_c color D4-branes and the resulting D4-branes stretch from the $NS5_{\theta}$ -brane to the NS5-brane and the coordinate of an intersection point between the k D4-branes and the NS5-brane is given by $(v, w) = (-v_{NS5_{\theta}}, 0)$ where $v_{NS5_{\theta}}$ is a distance between $v = 0$ and the v coordinate of k D4-branes. This corresponds to the k 's eigenvalues 0 of M (2.8). The remaining $(N'_c - k)$ flavor

D4-branes between the $NS5_\theta$ -brane and the $NS5'_L$ -brane are related to the corresponding eigenvalues of M (2.8), i.e., $\frac{m}{\alpha}\mathbf{1}_{N'_c-k}$. The coordinate of an intersection point between the $(N'_c - k)$ D4-branes and the $NS5'_L$ -brane is given by $(v, w) = (0, +v_{NS5_\theta} \cot \theta)$. The product $Y\tilde{Y}$ is given by (2.9) and the supersymmetric vacuum drawn in Figure 6B with $k = 0$ has the vacuum expectation values $Y = \tilde{Y} = 0$ and the gauge group $SU(\tilde{N}_c)$ is unbroken. The expectation value of M is given by $M = \frac{m}{\alpha}\mathbf{1}_{N'_c} = m\Lambda \cot \theta \mathbf{1}_{N'_c}$.

Then the magnetic superpotential can be written in terms of Φ through (2.10) and the classical supersymmetric vacua are given similarly. Now one splits the $(N'_c - k) \times (N'_c - k)$ block at the lower right corner of $h\Phi$ and $Y\tilde{Y}$ into blocks of size n and $(N'_c - k - n)$ as before. The full one loop potential for $\Phi_n, \varphi, \tilde{\varphi}$ can be described and the vacuum expectation value for $h\Phi_n$ is given by (2.12) with an appropriate number of color D4-branes. One obtains the eigenvalues for mass matrix for φ and $\tilde{\varphi}$ and the vacuum (2.12) is locally stable. The n flavor D4-branes of straight brane configuration of Figure 6B can bend due to the fact that there exists an attractive gravitational interaction between those flavor D4-branes and the NS5-brane from the DBI action.

One can move n D4-branes, from $(N'_c - k)$ D4-branes stretched between the $NS5'_L$ -brane and the $NS5_\theta$ -brane at $w = +v_{NS5_\theta} \cot \theta$, to the local minimum of the potential and the end points of these n D4-branes are at a nonzero w as in Figure 6B. The remaining $(N'_c - k - n)$ flavor D4-branes between the $NS5_\theta$ -brane and the $NS5'_L$ -brane are related to the corresponding eigenvalues of $h\Phi$ (2.11), i.e., $\frac{\mu^2}{\mu_\phi}\mathbf{1}_{N'_c-k-n}$. The coordinate of an intersection point between the $(N'_c - k - n)$ D4-branes and the $NS5'_L$ -brane is given by $(v, w) = (0, +v_{NS5_\theta} \cot \theta)$. The k D4-branes stretching from the $NS5_\theta$ -brane to the NS5-brane correspond to exactly the k 's eigenvalues 0 of $h\Phi$ (2.11). Finally, the remnant n “curved” flavor D4-branes between the $NS5_\theta$ -branes and the $NS5'_L$ -brane are related to the corresponding eigenvalues (2.12) of $h\Phi_n$. It can be computed that the local minimum occurs at $w \simeq \tan \theta \frac{y^4}{\ell_s^2 v_{NS5_\theta}}$ with $x^6 \equiv y$ [6].

4.3 Other electric and magnetic theories

The type IIA brane configuration [25] corresponding to $\mathcal{N} = 1$ supersymmetric gauge theory with gauge group (4.1) and the antisymmetric flavor for $SU(N_c)$, the conjugate symmetric flavor for $SU(N_c)$, eight fundamentals for $SU(N_c)$, a bifundamental X in the representation $(\mathbf{N}_c, \overline{\mathbf{N}}'_c)$ and its conjugate field \tilde{X} in the representation $(\overline{\mathbf{N}}_c, \mathbf{N}'_c)$, under the gauge group can be described similarly: a middle $NS5'_M$ -brane, the left $NS5'_L$ -brane, the right $NS5'_R$ -brane, the left $NS5_L$ -brane, the right $NS5_R$ -brane, N_c -, N'_c -color D4-branes, eight semi-infinite D6-branes, an $O6^+$ -plane and an $O6^-$ -plane. We take the arbitrary number of color D4-branes with the constraint $2N'_c \geq N_c - 4$. The bifundamentals X and \tilde{X} correspond to 4-4 strings

connecting the N_c -color D4-branes with N'_c -color D4-branes while the antisymmetric and conjugate symmetric flavors correspond to 4-4 strings connecting N_c D4-branes located at negative x^6 region and N_c D4-branes located at positive x^6 region.

The middle NS5'-brane is located at $x^6 = 0$ and the x^6 coordinates for the $NS5'_L$ -brane, $NS5_L$ -brane, $NS5_R$ -brane and $NS5'_R$ -brane are given by $x^6 = -y_2, -y_1, y_1$ and $x^6 = y_2$ respectively. The N_c D4-branes are suspended between the $NS5_L$ -brane(whose x^6 coordinate is given by $x^6 = -y_1$) and $NS5_R$ -brane(whose x^6 coordinate is given by $x^6 = y_1$) while the N'_c D4-branes are suspended between the $NS5'_L$ -brane and the $NS5_L$ -brane(and further they are suspended between the $NS5_R$ -brane and the $NS5'_R$ -brane). We draw this brane configuration in Figure 7A [25] for the vanishing mass for the bifundamentals¹⁰. The gauge couplings of $SU(N_c)$ and $SU(N'_c)$ are given by (4.2), as before.

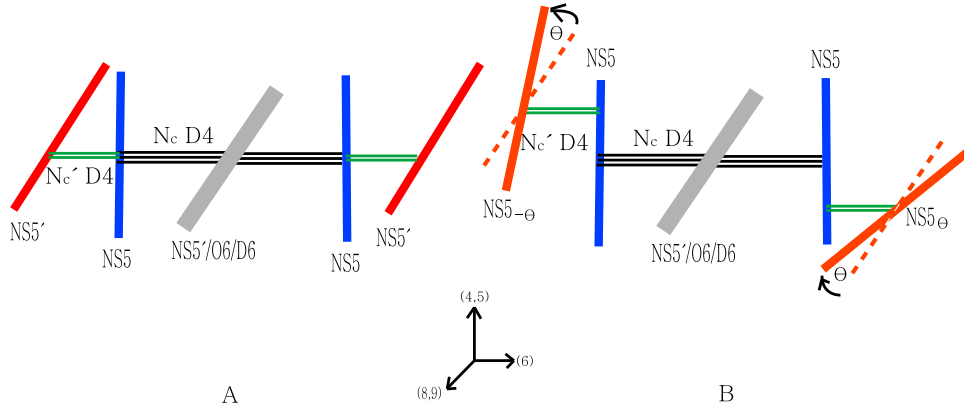


Figure 7: The $\mathcal{N} = 1$ supersymmetric electric brane configuration for the gauge group $SU(N_c) \times SU(N'_c)$ and the bifundamentals X and \tilde{X} as well as antisymmetric, conjugate symmetric flavors and eight D6-branes with vanishing mass term(7A) and nonvanishing mass and quartic terms(7B) for the bifundamentals. In Figure 7B, the new deformation by the rotation of $NS5'_L$ -brane by an angle $-\theta$ in (w, v) -plane arises(and its mirror by an angle θ).

There is no electric superpotential corresponding to the Figure 7A except the interaction term between the eight fundamental flavors and conjugate symmetric flavor. Now let us deform this theory. Displacing the two NS5'-branes relative each other in the v direction corresponds to turning on a quadratic superpotential for the bifundamentals X and \tilde{X} . The $NS5'_L$ -brane is moving to the $+v$ direction(and the $NS5'_R$ -brane is moving to $-v$ direction) due to the O6-plane for fixed NS5-branes. Moreover, the rotation of the $NS5'_L$ -brane with an angle $-\theta$ in (w, v) -plane leads to a quartic term for the bifundamentals. Then the deformed

¹⁰See also the relevant previous work appeared in [35, 21, 36] for supersymmetric vacua and [31, 32, 20] for nonsupersymmetric vacua. This brane realization is equivalent to the reduced brane configuration described in section 4 of [31] with particular rotations for the NS-branes if we remove all the D6-branes completely.

electric superpotential is given by (4.3) as well as an insertion of interaction term between the eight fundamental flavors and conjugate symmetric flavor [9].

We draw this brane configuration in Figure 7B for nonvanishing mass for the bifundamentals by moving the $NS5'_L$ -brane with N'_c color D4-branes to the $+v$ direction (and their mirrors to $-v$ direction) and rotating it by an angle $-\theta$ in (w, v) -plane. Compared with the brane configuration of [25], the rotation of $NS5'_L$ -brane is the only difference and $\theta \rightarrow 0$ limit reduces to the one of [25].

The solution for the supersymmetric vacua can be obtained by $X\tilde{X} = \frac{m}{\alpha}$ through the F-term conditions. This breaks the gauge group $SU(N_c) \times SU(N'_c)$ to $SU(N_c - k)$, $SU(N'_c - k)$, and $U(k)$. When the $NS5_L$ -brane moves to $+w$ direction, then the three NS-branes intersect in three points in (v, w) -plane. The coordinates of (v, w) for those points are $(v_{NS5-\theta}, v_{NS5-\theta} \cot \theta)$, $(0, v_{NS5-\theta} \cot \theta)$ and $(0, 2v_{NS5-\theta} \cot \theta)$. There exists the other intersection point given by $(-v_{NS5-\theta}, v_{NS5-\theta} \cot \theta)$. Then $(N_c - k)$ D4-branes are connecting between the $NS5_L$ -brane and the $NS5_R$ -brane. The $(N'_c - k)$ D4-branes are connecting between the $NS5_{-\theta}$ -brane and the $NS5_L$ -brane (and their mirrors). Finally, k D4-branes are connecting between the $NS5_{-\theta}$ -brane and the $NS5_\theta$ -brane directly. The distance from k D4-branes to the $NS5_L$ -brane can be read off from the trigonometric geometry and it is given by $w = +v_{NS5-\theta} \cot \theta$.

Let us apply the Seiberg dual to the $SU(N_c)$ factor. Starting from Figure 7B and moving the $NS5_L$ -brane to the right all the way past the $NS5'_M$ -brane (and $NS5_R$ -brane to the left of $NS5'_M$ -brane), one obtains the Figure 8A. By introducing N'_c D4-branes and N'_c anti-D4-branes between $NS5_R$ -brane and $NS5'_M$ -brane, we are left with $(N'_c - \tilde{N}_c)$ anti-D4-branes between $NS5_R$ -brane and $NS5'_M$ -brane (and its mirrors).

The gauge group is given by

$$SU(\tilde{N}_c = 2N'_c - N_c + 4) \times SU(N'_c) \quad (4.6)$$

where the number of dual color can be obtained from the linking number counting, as done in [31, 23]. The matter contents are the field Y in the bifundamental representation $(\tilde{\mathbf{N}}_c, \overline{\mathbf{N}}'_c)$ and its complex conjugate field \tilde{Y} in the bifundamental representation $(\overline{\tilde{\mathbf{N}}}_c, \mathbf{N}'_c)$, and the gauge singlet $X\tilde{X}$ in the representation for $(\mathbf{1}, \mathbf{N}'_c{}^2 - \mathbf{1}) \oplus (\mathbf{1}, \mathbf{1})$, under the dual gauge group. There are also the antisymmetric flavor a , the conjugate symmetric flavor \tilde{s} and eight fundamentals \hat{q} for $SU(\tilde{N}_c)$.

Then the dual magnetic superpotential, by adding the mass term and quartic term for the bifundamentals X and \tilde{X} , is given by

$$W_{dual} = \frac{1}{\Lambda} M Y \tilde{Y} - \frac{\alpha}{2} M^2 + m M + \hat{q} \tilde{s} \hat{q}. \quad (4.7)$$

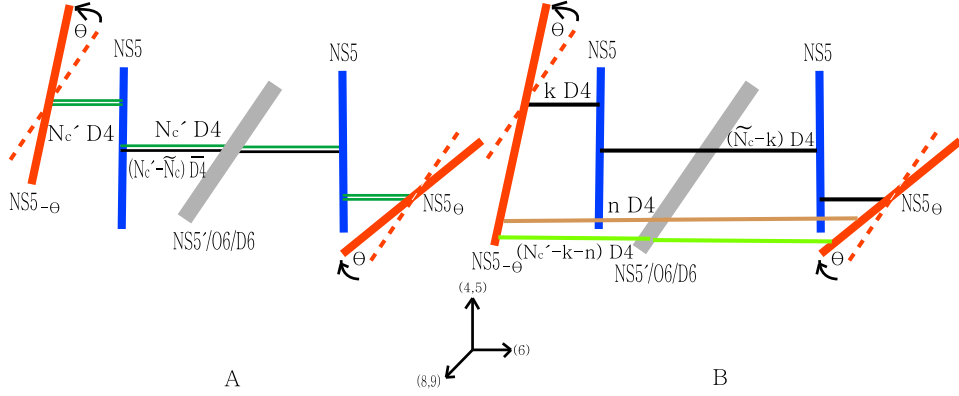


Figure 8: The magnetic brane configuration corresponding to Figure 7B with D4- and $\overline{D4}$ -branes(8A) when the distance between $NS5_{-\theta}$ -brane and the $NS5'$ -brane along v direction is large and with a misalignment between D4-branes(8B) when they are close to each other. The upper N'_c flavor D4-branes are splitting into $(N'_c - k)$ and k D4-branes. The location of intersection between the $NS5_{-\theta}$ -brane and the upper $(N'_c - k)$ D4-branes is given by $(v, w) = (0, +v_{NS5_{-\theta}} \cot \theta)$ while the one between the $NS5_{-\theta}$ -brane and the upper k D4-branes is given by $(v, w) = (+v_{NS5_{-\theta}}, 0)$. By moving n flavor D4-branes from the upper $(N'_c - k)$ flavor D4-branes, the nonzero positive w coordinate for n “curved” flavor D4-branes can be determined. Similarly, the location of intersection between the $NS5_{\theta}$ -brane and the lower $(N'_c - k)$ D4-branes is given by $(v, w) = (0, +v_{NS5_{\theta}} \cot \theta)$ while the one between the $NS5_{\theta}$ -brane and the lower k D4-branes is given by $(v, w) = (-v_{NS5_{\theta}}, 0)$ according to O6-plane action.

The brane configuration in Figure 8A is stable as long as the distance $v_{NS5_{-\theta}}$ between the upper $NS5_{-\theta}$ -brane and the middle $NS5'_M$ -brane is large. If they are close to each other then this brane configuration is unstable to decay and it becomes the brane configuration in Figure 8B. One can regard these brane configurations as particular states in the magnetic gauge theory with the gauge group (4.6) and superpotential (4.7). When the $NS5_{-\theta}$ -brane is replaced by N'_c coincident D6-branes, the brane configuration of Figure 8B is the same as the Figure 3 studied in [9]. This brane realization can be obtained from the Figure 2B by adding O6-plane with appropriate mirrors.

The conditions $b_{SU(\tilde{N}_c)}^{mag} < 0$ and $b_{SU(N_c)} > 0$ lead to $N'_c < \frac{2}{3}N_c - \frac{4}{3}$. Then the range for the N'_c can be written as $\frac{1}{2}N_c - 2 < N'_c < \frac{2}{3}N_c - \frac{4}{3}$. Moreover, $b_{SU(N'_c)} = 3N'_c - N_c > 0$ and $b_{SU(N'_c)}^{mag} = N_c - 4 > 0$. Then one can also analyze the hierarchy of scales previously.

In the brane configuration of Figure 8B, the k of the N'_c flavor D4-branes are connecting with k of \tilde{N}_c color D4-branes and the resulting D4-branes stretch from the $NS5_{\theta}$ -brane to the $NS5_L$ -brane and the coordinate of an intersection point between the k D4-branes and the $NS5_L$ -brane is given by $(v, w) = (-v_{NS5_{\theta}}, 0)$. The coordinate of an intersection point

between the $(N'_c - k)$ D4-branes and the NS5'-brane is given by $(v, w) = (0, +v_{NS5_\theta} \cot \theta)$.

Then the magnetic superpotential can be written in terms of Φ through

$$W_{dual} = h\Phi Y \tilde{Y} + \frac{h^2 \mu_\phi}{2} \text{tr} \Phi^2 - h\mu^2 \text{tr} \Phi + \hat{q} \tilde{s} \hat{q}$$

and the classical supersymmetric vacua are given similarly. The full one loop potential for $\Phi_n, \varphi, \tilde{\varphi}$ can be described and the vacuum expectation value for $h\Phi_n$ is given by (2.12). One obtains the eigenvalues for mass matrix for φ and $\tilde{\varphi}$ and the vacuum (2.12) is locally stable.

One can move n D4-branes, from $(N'_c - k)$ D4-branes stretched between the NS5'-brane and the $NS5_\theta$ -brane at $w = +v_{NS5_\theta} \cot \theta$, to the local minimum of the potential and the end points of these n D4-branes are at a nonzero w as in Figure 8B. The coordinate of an intersection point between the $(N'_c - k - n)$ D4-branes and the NS5'-brane is given by $(v, w) = (0, +v_{NS5_\theta} \cot \theta)$. The k D4-branes stretching from the $NS5_\theta$ -brane to the $NS5_L$ -brane correspond to exactly the k 's eigenvalues 0 of $h\Phi$. The local minimum occurs at $w \simeq \tan \theta \frac{y^4}{\ell_s^2 v_{NS5_\theta}}$ with $x^6 \equiv y$ [6] as before.

5 Conclusions and outlook

By following the spirit of [6, 7], when the quartic term for the bifundamentals in the superpotential is present, we have constructed the meta-stable brane configurations by rotating the NS-brane in (w, v) -plane from type IIA string theory. They are summarized by the Figures 2B, 4B, 6B and 8B.

As suggested in [6], it would be interesting to study what happens when we replace the NS5'-brane with other coincident D6-branes further. According to the discussion of [7], there exists a deformation from higher order terms for the quarks and it would be interesting to see how these terms can appear in the meta-stable brane configuration from type IIA string theory. It is an open problem to see how to construct a direct gauge mediation for our meta-stable vacua in the context of [37, 38].

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References

- [1] K. Intriligator, N. Seiberg and D. Shih, “Dynamical SUSY breaking in meta-stable vacua,” JHEP **0604**, 021 (2006) [arXiv:hep-th/0602239].
- [2] K. Intriligator and N. Seiberg, “Lectures on Supersymmetry Breaking,” Class. Quant. Grav. **24**, S741 (2007) [arXiv:hep-ph/0702069].
- [3] H. Ooguri and Y. Ookouchi, “Meta-stable supersymmetry breaking vacua on intersecting branes,” Phys. Lett. B **641**, 323 (2006) [arXiv:hep-th/0607183].
- [4] S. Franco, I. Garcia-Etxebarria and A. M. Uranga, “Non-supersymmetric meta-stable vacua from brane configurations,” JHEP **0701**, 085 (2007) [arXiv:hep-th/0607218].
- [5] I. Bena, E. Gorbatov, S. Hellerman, N. Seiberg and D. Shih, “A note on (meta)stable brane configurations in MQCD,” JHEP **0611**, 088 (2006) [arXiv:hep-th/0608157].
- [6] A. Giveon and D. Kutasov, “Stable and Metastable Vacua in Brane Constructions of SQCD,” arXiv:0710.1833 [hep-th].
- [7] A. Giveon and D. Kutasov, “Stable and Metastable Vacua in SQCD,” arXiv:0710.0894 [hep-th].
- [8] C. Ahn, “Other Meta-Stable Brane Configuration by Adding an Orientifold 6-Plane to Giveon-Kutasov,” arXiv:0712.0032 [hep-th].
- [9] C. Ahn, “More on Meta-Stable Brane Configuration by Quartic Superpotential for Fundamentals,” arXiv:0801.0257 [hep-th].
- [10] A. Giveon and D. Kutasov, “Brane dynamics and gauge theory,” Rev. Mod. Phys. **71**, 983 (1999) [arXiv:hep-th/9802067].
- [11] A. Giveon and D. Kutasov, “Gauge symmetry and supersymmetry breaking from intersecting branes,” Nucl. Phys. B **778**, 129 (2007) [arXiv:hep-th/0703135].
- [12] C. Ahn, “Brane configurations for nonsupersymmetric meta-stable vacua in SQCD with adjoint matter,” Class. Quant. Grav. **24**, 1359 (2007) [arXiv:hep-th/0608160].
- [13] C. Ahn, “M-theory lift of meta-stable brane configuration in symplectic and orthogonal gauge groups,” Phys. Lett. B **647**, 493 (2007) [arXiv:hep-th/0610025].

- [14] J. H. Brodie and A. Hanany, “Type IIA superstrings, chiral symmetry, and $N = 1$ 4D gauge theory dualities,” Nucl. Phys. B **506**, 157 (1997) [arXiv:hep-th/9704043].
- [15] C. Ahn and R. Tatar, “Geometry, D-branes and $N = 1$ duality in four dimensions with product gauge groups,” Phys. Lett. B **413**, 293 (1997) [arXiv:hep-th/9705106].
- [16] K. A. Intriligator, R. G. Leigh and M. J. Strassler, “New examples of duality in chiral and nonchiral supersymmetric gauge theories,” Nucl. Phys. B **456**, 567 (1995) [arXiv:hep-th/9506148].
- [17] E. Barnes, K. Intriligator, B. Wecht and J. Wright, “ $N = 1$ RG flows, product groups, and a-maximization,” Nucl. Phys. B **716**, 33 (2005) [arXiv:hep-th/0502049].
- [18] C. Ahn, “Meta-stable brane configuration of product gauge groups,” arXiv:0704.0121 [hep-th].
- [19] C. Ahn, “Meta-Stable Brane Configurations of Triple Product Gauge Groups,” arXiv:0708.4255 [hep-th].
- [20] C. Ahn, “Meta-Stable Brane Configurations with Multiple NS5-Branes,” arXiv:0711.0082 [hep-th].
- [21] I. Brunner, A. Hanany, A. Karch and D. Lust, “Brane dynamics and chiral non-chiral transitions,” Nucl. Phys. B **528**, 197 (1998) [arXiv:hep-th/9801017].
- [22] C. Ahn, “Meta-stable brane configuration with orientifold 6 plane,” JHEP **0705**, 053 (2007) [arXiv:hep-th/0701145].
- [23] C. Ahn, “More on Meta-Stable Brane Configuration,” Class. Quant. Grav. **24**, 3603 (2007) [arXiv:hep-th/0702038].
- [24] A. Amariti, L. Girardello and A. Mariotti, “Meta-stable A_n quiver gauge theories,” JHEP **0710**, 017 (2007) [arXiv:0706.3151 [hep-th]].
- [25] C. Ahn, “Meta-Stable Brane Configurations by Adding an Orientifold-Plane to Giveon-Kutasov,” JHEP **0708**, 021 (2007) [arXiv:0706.0042 [hep-th]].
- [26] C. Ahn, “Meta-Stable Brane Configuration and Gauged Flavor Symmetry,” Mod. Phys. Lett. A **22**, 2329 (2007) [arXiv:hep-th/0703015].
- [27] R. Tatar, “Dualities in 4D theories with product gauge groups from brane configurations,” Phys. Lett. B **419**, 99 (1998) [arXiv:hep-th/9704198].

- [28] C. Ahn, K. Oh and R. Tatar, “Branes, geometry and $N = 1$ duality with product gauge groups of SO and Sp ,” J. Geom. Phys. **31**, 301 (1999) [arXiv:hep-th/9707027].
- [29] J. Hashiba, “Branes and vector-like supersymmetry breaking theories with gauged global symmetry,” Nucl. Phys. B **550**, 329 (1999) [arXiv:hep-th/9809181].
- [30] K. Landsteiner, E. Lopez and D. A. Lowe, “Supersymmetric gauge theories from branes and orientifold six-planes,” JHEP **9807**, 011 (1998) [arXiv:hep-th/9805158].
- [31] C. Ahn, “Meta-stable brane configurations with five NS5-branes,” arXiv:0705.0056 [hep-th].
- [32] C. Ahn, “Meta-Stable Brane Configurations of Multiple Product Gauge Groups with Orientifold 6 Plane,” arXiv:0710.0180 [hep-th].
- [33] C. Ahn, “More Meta-Stable Brane Configurations without D6-Branes,” Nucl. Phys. B **790**, 281 (2008) [arXiv:0707.0092 [hep-th]].
- [34] C. Ahn, “Meta-Stable Brane Configurations with Seven NS5-Branes,” arXiv:0708.0439 [hep-th].
- [35] K. Landsteiner, E. Lopez and D. A. Lowe, “Duality of chiral $N = 1$ supersymmetric gauge theories via branes,” JHEP **9802**, 007 (1998) [arXiv:hep-th/9801002].
- [36] S. Elitzur, A. Giveon, D. Kutasov and D. Tsabar, “Branes, orientifolds and chiral gauge theories,” Nucl. Phys. B **524**, 251 (1998) [arXiv:hep-th/9801020].
- [37] N. Haba and N. Maru, “A Simple Model of Direct Gauge Mediation of Metastable Supersymmetry Breaking,” arXiv:0709.2945 [hep-ph].
- [38] F. Xu and J. M. Yang, “An Extension for Direct Gauge Mediation of Metastable Supersymmetry arXiv:0712.4111 [hep-ph].